Kinetic molecular model of a gas: postulates and derivation of the kinetic gas equation

Content Type	Notes
Status	Done
Chapter	Gaseous State
Course	Graduation
Reference	
Subject	Physical Chemistry
Topic	Kinetic Theory
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Progress (%)	100
Difficulty Level	Medium
Mistake type	
Previous Year Questions	

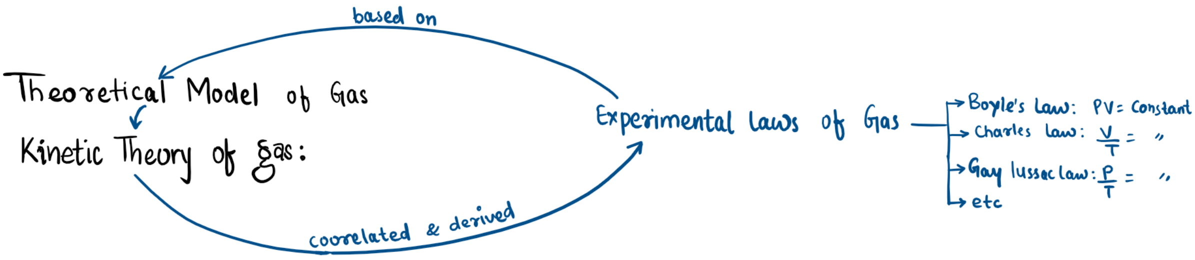
Doubt: Difference between kinetic molecular model vs kinetic theory of gases

Both terms are used to describe the same scientific theory, so there is no significant difference between the two.

Theoretical Model of gas

Kinetic Theory of Gas

The kinetic molecular theory of gases is a theoretical model that explains the macroscopic behaviour of gases in terms of the behaviour and interactions of their constituent particles (atoms or molecules).



How is kinetic Theory related to experimental laws?

The kinetic theory of gases is related to several experimental laws that describe the behavior of gases. These include:

1. Boyle's Law: This states that the pressure of a gas is inversely proportional to its volume, at constant temperature. This relationship can be derived from the kinetic theory by considering the average force exerted by the gas particles on the walls of the container.
2. Charles' Law: This states that the volume of a gas is directly proportional to its temperature, at constant pressure. This relationship can be derived from the kinetic theory by considering the effect of temperature on the average velocity of the gas particles.
3. Avogadro's Law: This states that the volume of a gas is directly proportional to the number of particles it contains, at constant temperature and pressure. This relationship can be derived from the kinetic theory by considering the effect of the number of particles on the pressure of the gas.

These experimental laws are all special cases of the Ideal Gas Law, which states that the pressure, volume, and temperature of a gas are related by:

$PV = nRT$,

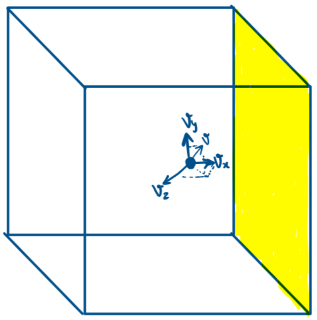
where n is the number of moles of gas, R is the gas constant, and T is the temperature in kelvin. This equation is based on the kinetic theory of gases and incorporates the relationships described by Boyle's Law, Charles' Law, and Avogadro's Law.

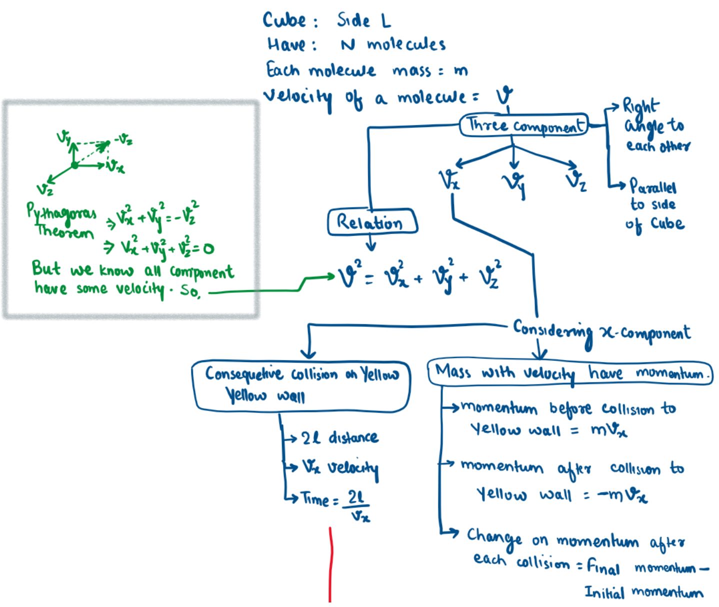
Postulates of kinetic theory

Postulates (Applicable For Ideal Gas) :

- Negligible gas particle's volume
- No Force of attraction between particles
  - Moving independently.
  - Random Motion
- Collision between particles is perfectly elastic
  - No energy loss.
- Laws of Classical Mechanics are applicable.
  - mainly: Second Law of Motion.
- Kinetic energy
  - At an instant & for a given molecule : Energy from Large to very small possible
  - Average kinetic Energy = Constant (At a particular temperature)
    - $\boxed{(KE)_{Average} \propto \text{Temperature}}$

Derivation of kinetic gas equation





Cube : Side L

Have : N molecules

Each molecule mass : m

Velocity of a molecule : v

Three component

- Right angle to each other
- Parallel to side of Cube

v_x v_y v_z

Pythagoras Theorem $\Rightarrow v_x^2 + v_y^2 = -v_z^2$

$\Rightarrow v_x^2 + v_y^2 + v_z^2 = 0$

But we know all component have some velocity. So,

Relation

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

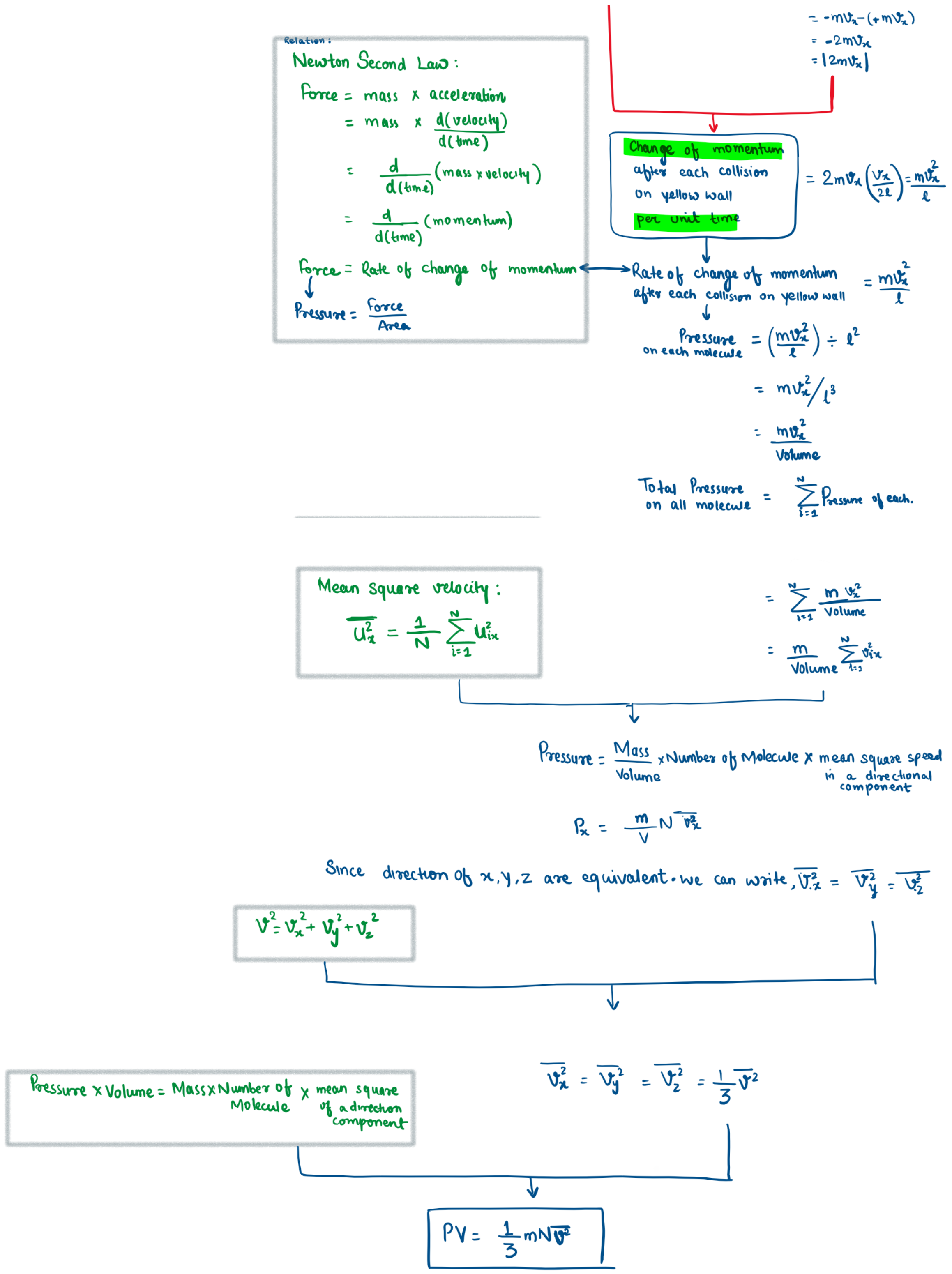
Considering x-component

Consequetive collision on Yellow Yellow wall

- $2l$ distance
- v_x velocity
- $\text{Time} = \frac{2l}{v_x}$

Mass with velocity have momentum.

- momentum before collision to Yellow wall $= mv_x$
- momentum after collision to Yellow wall $= -mv_x$
- Change on momentum after each collision = Final momentum − Initial momentum


$= -mv_x - (+mv_x)$
$= -2mv_x$
$= |2mv_x|$
Relation:
Newton Second Law:
Force = mass x acceleration
= mass x $\frac{d(\text{velocity})}{d(\text{time})}$
= $\frac{d}{d(\text{time})}$(mass x velocity)
= $\frac{d}{d(\text{time})}$(momentum)
Force = Rate of change of momentum
Pressure = $\frac{\text{Force}}{\text{Area}}$
Change of momentum after each collision on yellow wall per unit time
$= 2mv_x\left(\frac{v_x}{2l}\right) = \frac{mv_x^2}{l}$
Rate of change of momentum after each collision on yellow wall
$= \frac{mv_x^2}{l}$
Pressure on each molecule $= \left(\frac{mv_x^2}{l}\right) \div l^2$
$= mv_x^2/l^3$
$= \frac{mv_x^2}{\text{Volume}}$
Total Pressure on all molecule $= \sum_{i=1}^{N}$ Pressure of each.
$= \sum_{i=1}^{N} \frac{m v_x^2}{\text{Volume}}$
$= \frac{m}{\text{Volume}} \sum_{i=1}^{N} v_{ix}^2$
Mean square velocity:
$\overline{u_x^2} = \frac{1}{N}\sum_{i=1}^{N} u_{ix}^2$
Pressure = $\frac{\text{Mass}}{\text{Volume}}$ x Number of Molecule x mean square speed in a directional component
$P_x = \frac{m}{V} N \overline{v_x^2}$
Since direction of x,y,z are equivalent. we can write, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$
$v^2 = v_x^2 + v_y^2 + v_z^2$
$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2}$
Pressure x Volume = Mass x Number of Molecule x mean square of a direction component
$PV = \frac{1}{3} mN\overline{v^2}$

Steps for the derivation of kinetic gas equation

Here's one way to do the derivation:

1. Consider a gas confined to a container of volume V.

2. The pressure of the gas is proportional to the force exerted by the gas particles on the walls of the container.
3. The force exerted by each particle on the walls can be calculated as the product of the particle's velocity and its mass.
4. The average velocity of the particles can be found using the Maxwell-Boltzmann distribution, which describes the velocity distribution of particles in a gas.
5. Integrating the velocity distribution over all velocities gives the average velocity of the particles, which can then be used to find the average force exerted by each particle on the walls of the container.
6. Finally, the pressure of the gas can be calculated by dividing the total force by the area of the container.
7. When this is done, it can be shown that the pressure of an ideal gas is proportional to the temperature and the number of particles:

P = (NkT)/V,

where N is the number of particles, k is the Boltzmann constant, T is the temperature in kelvin, and V is the volume.

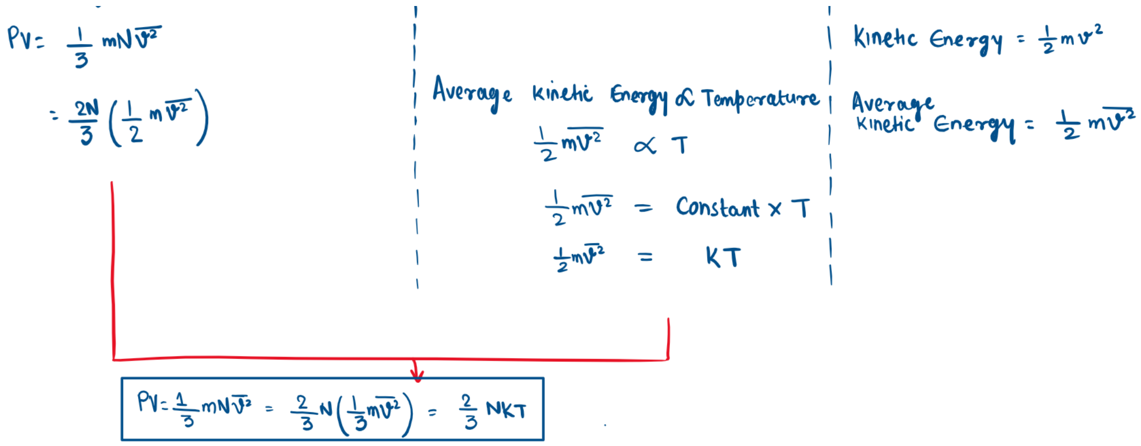
Kinetic Gas Equation

The Kinetic Gas Equation is a fundamental equation in statistical mechanics that relates the pressure, temperature, and number of particles in an ideal gas. It states that the pressure of an ideal gas is proportional to the average kinetic energy of its particles.

$$\text{Pressure} \times \text{Volume} = \frac{1}{3} \text{ of Mass} \times \text{Number of molecules} \times \text{Mean Square velocity}$$

$$PV = \frac{1}{3} m N \overline{v^2}$$

Kinetic Gas Equation involving kelvin temperature


$PV = \frac{1}{3} m N \overline{v^2}$
$= \frac{2N}{3}\left(\frac{1}{2} m \overline{v^2}\right)$
Average kinetic Energy ∝ Temperature
$\frac{1}{2} m \overline{v^2} \propto T$
$\frac{1}{2} m \overline{v^2} = \text{Constant} \times T$
$\frac{1}{2} m \overline{v^2} = KT$
Kinetic Energy $= \frac{1}{2} m v^2$
Average Kinetic Energy $= \frac{1}{2} m \overline{v^2}$
$PV = \frac{1}{3} m N \overline{v^2} = \frac{2}{3} N \left(\frac{1}{3} m \overline{v^2}\right) = \frac{2}{3} NKT$

⬆️ It is important to note that the temperature T in the equation is in kelvin, not in Celsius or Fahrenheit.

Why kelvin and not celsius?

The kelvin temperature scale is used in the kinetic gas equation because it is an absolute temperature scale, meaning that it starts at absolute zero, the temperature at which all matter has no thermal energy. In contrast, the Celsius temperature scale is a relative temperature scale, meaning that it is based on a reference temperature, such as the freezing point of water.

The use of an absolute temperature scale is important in physics and thermodynamics because it allows for the calculation of the average kinetic energy of particles, which is a fundamental property that depends only on the temperature of the system. Using an absolute temperature scale also ensures that temperature has a well-defined meaning and can be compared between different systems and states of matter.

Therefore, when studying the behaviour of an ideal gas, it is more convenient to use the kelvin temperature scale, as it provides a well-defined and absolute measure of temperature, which is essential for the calculation of the pressure of the gas.

A common way to remember the Kinetic gas equation

PV = NkT,

which can be interpreted as "Pressure times Volume is equal to the number of particles multiplied by the Boltzmann constant and the temperature."